

# W = emgr(f,g,s,t,w,pr,nf,ut,us,xs,um,xm,dp)

emgr – EMpirical GRamian Framework ( Version 5.9 )

## Mandatory Arguments

<b>f</b>	System Vector Field	(Handle) $x = f(x,u,p,t)$	i.e.: $f = @(x,u,p,t) A*x+B*u+F*p$
<b>g</b>	Output Functional	(Handle) $y = g(x,u,p,t)$	i.e.: $g = @(x,u,p,t) C*x+D*u$
		<b>1</b> $y = x$	
<b>s</b>	System Dimensions	(Vector) $s = [M,N,Q]$	(Inputs, States, Outputs)
<b>t</b>	Time Discretization	(Vector) $t = [dt,Tf]$	(Time Step, Time Horizon)
<b>w</b>	Gramian Type	(Char)	Empirical System Gramian Type
		'c'	Empirical Controllability Gramian (returns $W_c$ )
		'o'	Empirical Observability Gramian (returns $W_o$ )
		'x'	Empirical Cross Gramian (returns $W_x$ )
		'y'	Empirical Linear Cross Gramian (returns $W_y$ )
		's'	Empirical Sensitivity Gramian (returns $\{W_c, W_s\}$ )
		'i'	Empirical Identifiability Gramian (returns $\{W_o, W_i\}$ )
		'j'	Empirical Joint Gramian (returns $\{W_x, W_j\}$ )

## Optional Arguments

<b>pr</b>	Parameters	(Vector)	Column vector of parameters (default: $pr = 0$ )
		(Matrix)	Set of parameter columns ( $W_s, W_i, W_j$ require min & max)
<b>nf</b>	Options Flags	(Vector)	Thirteen components (default: $nf = 0$ )
<b>ut</b>	Input Function	(Handle)	Input function $u_t = ut(t)$ or char (default: $ut = 'i'$ )
		'i'	Delta impulse input (default)
		's'	Step input
		'h'	Haversine decaying exponential chirp input
		'a'	Cardinale sine input
		'r'	Pseudo-random binary input
<b>us</b>	Steady-State Input	(Scalar)	Uniform steady-state input (default: $us = 0$ )
		(Vector)	Individual steady-state input ( $M \times 1$ )
<b>xs</b>	Steady-State	(Scalar)	Uniform steady-state (default: $xs = 0$ )
		(Vector)	Individual steady-states ( $N \times 1$ )
<b>um</b>	Input Scales	(Scalar)	Uniform max input scales (default: $um = 1$ )
		(Vector)	Individual max input scales ( $M \times 1$ )
		(Matrix)	Custom input scales ( $M \times *$ )
<b>xm</b>	Steady-State Scales	(Scalar)	Uniform max steady-state scales (default: $xm = 1$ )
		(Vector)	Individual max steady-state scales ( $N \times 1$ )
		(Matrix)	Custom steady-state scales ( $N \times *$ )
<b>dp</b>	Dot Product	(Handle)	Custom Inner product / kernel $xy = dp(x,y)$ , (default: $dp = []$ )

## Option Flags

<b>nf(1)</b>	Trajectory Centering		
	0	None (default)	
	1	Steady state	
	2	Final state	
	3	Arithmetic average	
	4	Root-mean-squared	
	5	Mid-range	
<b>nf(2)</b>	Input Scale Sequence		
	0	Single (default)	
	1	Linear	
	2	Geometric	
	3	Logarithmic	
	4	Sparse	
<b>nf(3)</b>	State Scale Sequence		
	0	Single (default)	
	1	Linear	
	2	Geometric	
	3	Logarithmic	
	4	Sparse	
<b>nf(4)</b>	Input Transformation		
	0	$\pm$ Unit (default)	
	1	+ Unit	
<b>nf(5)</b>	State Transformation		
	0	$\pm$ Unit (default)	
	1	+ Unit	
<b>nf(6)</b>	Normalizing		
	0	None (default)	
	1	Steady state	
	2	Jacobi	
<b>nf(7)</b>	State Gramian Type		
	0	Regular (default)	
$W_c, W_s$	1	Output controllability Gramian	
$W_o, W_i$	1	Averaged observability Gramian	
$W_x, W_y, W_j$	1	Non-symmetric cross Gramian	
<b>nf(8)</b>	Extra Input ( $W_o, W_x, W_s, W_i, W_j$ only)		
	0	No (default)	
	1	Yes	
<b>nf(9)</b>	Center Parameter Scales ( $W_s, W_i, W_j$ only)		
	0	None (default)	
	1	Linear mean centering	
	2	Logarithmic mean centering	
<b>nf(10)</b>	Parameter Gramian Type ( $W_s, W_i, W_j$ only)		
$W_s$	0	Input-state average (default)	
$W_s$	1	Input-output average	
$W_i, W_j$	0	Approx. Schur-complement (default)	
$W_i, W_j$	1	Coarse Schur-complement	
<b>nf(11)</b>	Partitioned Cross Gramian ( $W_x, W_j$ only)		
	0	Full cross Gramian (default)	
	<N	Cross Gramian partition size	
<b>nf(12)</b>	Partitioned Cross Gramian ( $W_x, W_j$ only)		
	0	Full cross Gramian (default)	
	>0	Partition running index	
<b>nf(13)</b>	Trajectory Weighting		
	0	None (default)	
	1	Linear time-weighting	
	2	Quadratic time-weighting	
	3	State weighting	
	4	Scale weighting	

## Custom Solver

Global variable **ODE** is a handle with signature:  
 $y = ODE(f,g,t,x0,u,p)$  (default: RK – SSP32)

**About Info:**  $V = emgr('version')$

**More info at:** <https://gramian.de>