

# W = emgr (f, g, s, t, w, pr, nf, ut, us, xs, um, xm, dp) ;

## emgr – Empirical Gramian Framework ( Version 5.3 )

### Mandatory Arguments

<b>f</b>	System Vector Field	(Handle)	$\mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t)$	i.e.: $\mathbf{f} = @(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) \mathbf{A}*\mathbf{x} + \mathbf{B}*\mathbf{u} + \mathbf{F}*\mathbf{p}$ ;
<b>g</b>	Output Functional	(Handle)	$\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t)$	i.e.: $\mathbf{g} = @(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) \mathbf{C}*\mathbf{x} + \mathbf{D}*\mathbf{u}$ ;
		1	$\mathbf{y} = \mathbf{x}$	
<b>s</b>	System Dimensions	(Vector)	$\mathbf{s} = [\mathbf{M}, \mathbf{N}, \mathbf{Q}]$	(Inputs, States, Outputs)
<b>t</b>	Time	(Vector)	$\mathbf{t} = [\mathbf{h}, \mathbf{T}]$	(Time Step, Time Horizon)
<b>w</b>	Gramian Type	(Character)		
		'c'	Empirical Controllability Gramian (returns <b>Wc</b> )	
		'o'	Empirical Observability Gramian (returns <b>Wo</b> )	
		'x'	Empirical Cross Gramian (returns <b>Wx</b> )	
		'y'	Empirical Linear Cross Gramian (returns <b>Wy</b> )	
		's'	Empirical Sensitivity Gramian (returns <b>{Wc,Ws}</b> )	
		'i'	Empirical Identifiability Gramian (returns <b>{Wo,Wi}</b> )	
		'j'	Empirical Joint Gramian (returns <b>{Wx,Wj}</b> )	

### Optional Arguments

<b>pr</b>	Parameters	(Vector)	Column Vector of System Parameters (Default: <b>pr = 0</b> )
		(Matrix)	Set of Parameter Column Vectors ('s','i','j' requires two)
<b>nf</b>	Options	(Vector)	Twelve Components (Default: <b>nf = 0</b> ), see Option Flags
<b>ut</b>	Input Function	(Handle)	Input function $\mathbf{u} = \mathbf{ut}(\mathbf{t})$ (Default: <b>ut = 1</b> )
		1	Delta Impulse Input
		$\infty$	Linear Chirp Function
<b>us</b>	Steady-State Input	(Scalar)	Uniform Steady-State Input (Default: <b>us = 0</b> )
		(Vector)	Individual Steady-State Input ( <b>J x 1</b> )
<b>xs</b>	Steady State	(Scalar)	Uniform Steady State (Default: <b>xs = 0</b> )
		(Vector)	Individual Steady States ( <b>N x 1</b> )
<b>um</b>	Input Scales	(Scalar)	Uniform Maximum Input Scales (Default: <b>um = 1</b> )
		(Vector)	Individual Maximum Input Scales ( <b>J x 1</b> )
		(Matrix)	Custom Input Scales ( <b>J x *</b> )
<b>xm</b>	Steady-State Scales	(Scalar)	Uniform Maximum Steady-State Scales (Default: <b>xm = 1</b> )
		(Vector)	Individual Maximum Steady-State Scales ( <b>N x 1</b> )
		(Matrix)	Custom Steady-State Scales ( <b>N x *</b> )
<b>dp</b>	Dot Product	(Handle)	Handle to custom inner product $\mathbf{z} = \mathbf{dp}(\mathbf{x}, \mathbf{y})$
		1	Default Matrix product

### Option Flags

<b>nf (1)</b>	Trajectory Centering		
	0	None (Default)	
	1	Initial State	
	2	Final Steady State	
	3	Arithmetic Average	
	4	Root-Mean-Squared	
	5	Midrange	
<b>nf (2)</b>	Input Scale Sequence		
	0	Single (Default)	
	1	Linear	
	2	Geometric	
	3	Logarithmic	
	4	Sparse	
<b>nf (3)</b>	State Scale Sequence		
	0	Single (Default)	
	1	Linear	
	2	Geometric	
	3	Logarithmic	
	4	Sparse	
<b>nf (4)</b>	Input Transformation		
	0	Unit (Default)	
	1	Inverse	
<b>nf (5)</b>	State Transformation		
	0	Unit (Default)	
	1	Inverse	
<b>nf (6)</b>	Normalizing ( <b>Wc,Wo,Wx,Wy</b> only)		
	0	None (Default)	
	1	Jacobi	
	2	Steady-State	
<b>nf (7)</b>	Non-Symmetric Cross Gramian ( <b>Wx,Wy,Wj</b> only)		
	0	Off (Default)	
	1	Non-Sym. Cross Gramian	
<b>nf (8)</b>	Extra Input ( <b>Wo,Wx,Ws,Wi,Wj</b> only)		
	0	No Extra Input (Default)	
	1	Parameter Perturbation Only	
<b>nf (9)</b>	Center Param. Scales ( <b>Ws,Wi,Wj</b> only)		
	0	No Centering	
	1	Arithmetic Mean Centering	
	2	Logarithmic Mean Centering	
<b>nf (10)</b>	Schur complement Options ( <b>Wi,Wj</b> only)		
	0	Detailed Schur-Complement (Default)	
	1	Approximate Schur-Complement	
<b>nf (11)</b>	Partitioned cross Gramian ( <b>Wx,Wj</b> only)		
	0	Full cross Gramian	
	<N	Cross Gramian Partition Size	
<b>nf (12)</b>	Partitioned cross Gramian ( <b>Wx,Wj</b> only)		
	0	Full cross Gramian	
	>0	Partition running index	

### Custom Solver

Global variable **ODE** to handle with signature:  
**y = solver(f,g,t,x,u,p)**; Default: RK - SSP32

**Minimal Usage: W = emgr(f,g,s,t,w);**

**About Info: V = emgr('version');**

More info at: <http://gramian.de>